

Accelerating internal dimensions and nonzero positive cosmological constant

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Abstract

We present a new scenario for the moduli stabilization and a very small but nonzero positive cosmological constant λ . In this scenario the complex structure moduli are still stabilized by the three-form fluxes as in the usual flux compactifications, but the Kähler modulus is not fixed by the KKLT scenario. In our case the scale factor of the internal dimensions is basically allowed to change with time. But at the supergravity level it is fixed by a set of dynamical (plus constraint) equations defined on the 4D spacetime, not by the nonperturbative corrections of KKLT. Also at the supergravity level it is shown that λ is fine-tuned to zero, $\lambda = 0$, by the same set of 4D equations. This result changes once we admit α' -corrections of the string theory. The fine-tuning $\lambda = 0$ changes into $\lambda = \frac{2}{3}Q$, where Q is a constant representing quantum corrections of the 6D action defined on the internal dimensions and its value is determined by the α' -corrections. It is also shown that this nonzero λ must be positive and at the same time the internal dimensions must evolve with time almost at the same rate as the external dimensions in the case of nonzero λ .

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I. Introduction

It is known that the three-dimensional space of our present universe is now under accelerated expansion [1], which means that the background vacuum of our present universe has its own energy density called dark energy, or the cosmological constant in the conventional sense. The cosmological constant λ is associated with quantum fluctuations of our vacuum and it must have some positive value to generate the accelerated expansion described above. Indeed, observations show that λ takes a positive value as mentioned above, but the mystery is that it is unreasonably too small as compared with the theoretical value calculated from the quantum theory, and this leads to a hierarchy problem called cosmological constant problem.

There have been many attempts to address this problem (for the review, see for instance [2]), but it has remained as an unsolved problem. But very recently, a new mechanism has been proposed to address this problem [3], which is very distinguished from the conventional theories where λ is directly determined from the scalar potential $\mathcal{V}_{\text{scalar}}$. In this mechanism λ contains a supersymmetry breaking term \mathcal{E}_{SB} besides the usual $\mathcal{V}_{\text{scalar}}$ of the $N = 1$ supergravity and where \mathcal{E}_{SB} has its own gauge arbitrariness. Thus the nonzero contributions to $\mathcal{V}_{\text{scalar}}$ coming from the perturbative and nonperturbative corrections, and also the NS-NS and R-R vacuum energies on the branes arising from quantum fluctuations are all gauged away by \mathcal{E}_{SB} (and by a certain self-tuning mechanism) and as a result λ is fine-tuned to vanish. In this self-tuning mechanism, whether λ vanishes or not is basically determined by the tensor structure of $\mathcal{V}_{\text{scalar}}$, not by the zero or nonzero values of $\mathcal{V}_{\text{scalar}}$ itself. In [3], the above self-tuning mechanism has been applied to the well-known KKLT model [4] to address the cosmological constant problem, especially aiming at explaining the vanishing λ of our present universe.

In KKLT, the geometry (or the complex structure moduli) of the internal dimensions is stabilized by the three-form fluxes as in the usual flux compactifications, but the scale factor (or the Kähler moduli) of the internal dimensions is fixed by a certain KKLT mechanism in which the scalar potential acquires a minimum point by a Kähler modulus-dependent nonperturbative correction. In the present paper we want to consider the self-tuning mechanism proposed in [3] again. But this time we do not apply it to the KKLT. In the present paper we assume that the complex structure moduli are still stabilized by the three-form fluxes. But the scale factor of the internal dimensions is not fixed by the KKLT scenario. In our present paper we basically assume that the internal dimensions are allowed to evolve with time. But nevertheless, we show that the scale factor of the internal dimensions is fixed at the supergravity level by a set of 4D equations, not by the Kähler modulus-dependent nonperturbative corrections of KKLT, in the simplest

setup. So in our model the no-scale structure is unbroken as in Ref. [5].

In this rather unconventional model λ is fine-tuned to zero as in [3], again at the supergravity level. But once we admit α' -corrections of the string theory, the fine-tuning $\lambda = 0$ changes into $\lambda = \frac{2}{3}Q$, where Q is a constant representing quantum corrections of the 6D action defined on the internal dimensions and its value is determined by the α' -corrections. Namely λ acquires nonzero values from the α' -corrections. In Sec. 10.2 we will show that this nonzero λ must be positive and at the same time the internal dimensions must evolve with time almost at the same rate as the external dimensions in the case of nonzero λ . In this paper we aim at explaining both of these two aspects of λ and the internal dimensions, based on the self-tuning mechanism presented in [3].

II. Time-dependent metric of the internal dimensions

In the string frame the type IIB action is given by

$$I_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-G_{10}} \left\{ e^{-2\Phi} [\mathcal{R}_{10} + 4(\nabla\Phi)^2] - \frac{1}{2 \cdot 3!} G_{(3)} \cdot \bar{G}_{(3)} - \frac{1}{4 \cdot 5!} \tilde{F}_{(5)}^2 \right\} \\ + \frac{1}{8i\kappa_{10}^2} \int e^{\Phi} A_{(4)} \wedge G_{(3)} \wedge \bar{G}_{(3)} , \quad (2.1)$$

where $G_{(3)} = F_{(3)} - ie^{-\Phi} H_{(3)}$ and $\tilde{F}_{(5)}$ is given by $\tilde{F}_{(5)} = F_{(5)} - \frac{1}{2} A_{(2)} \wedge H_{(3)} + \frac{1}{2} B_{(2)} \wedge F_{(3)}$ with $F_{(n+1)} = dA_{(n)}$ etc. In (2.1), we have omitted the one-form field strength term $F_{(1)}^2$ of the axion $A_{(0)}$ because, unlikely in the theories with scalar fields like quintessence, the axion does not play any important role in our discussions of this paper. But in our paper we basically consider the case where the three-form fluxes take nonzero values and the complex structure moduli of the internal dimensions are stabilized by these three-form fluxes. But this does not mean that we restrict our discussions only to the flux compactifications. Our discussions of this paper can be applied to both of the flux compactifications with $G_{(3)} \neq 0$ and the conventional compactifications with $G_{(3)} = 0$.¹

Now we introduce an ansatz for the 10D metric as

$$ds_{10}^2 = \alpha^2(\hat{t}) e^{A(y)} \hat{g}_{\mu\nu}(\hat{x}) d\hat{x}^\mu d\hat{x}^\nu + \beta^2(\hat{t}) e^{B(y)} h_{mn}(y) dy^m dy^n , \quad (2.2)$$

where $\hat{g}_{\mu\nu}(\hat{x})$ is the metric of the 4D spacetime,

$$\hat{g}_{\mu\nu}(\hat{x}) d\hat{x}^\mu d\hat{x}^\nu = -d\hat{t}^2 + a^2(\hat{t}) d\vec{x}_3^2 , \quad (2.3)$$

¹There is a different viewpoint on the moduli stabilization which does not use the usual flux compactifications. For instance, in Sec. III of Ref. [6] it was argued that the Calabi-Yau threefolds may be thought of as NS-NS solitons whose ADM masses are proportional to $1/g_s^2$. Hence in the limit $g_s \rightarrow 0$, these Calabi-Yau threefolds are very heavy and rigid and consequently deformations of internal geometry are highly suppressed.

while $h_{mn}(y)$ represents the metric of the 6D internal dimensions. In (2.2), $\alpha^2(\hat{t})$ is an extra degree of freedom which could have been absorbed into $\hat{g}_{\mu\nu}(\hat{x})d\hat{x}^\mu d\hat{x}^\nu$ by the coordinate transformation $d\hat{t} \rightarrow dt \equiv \alpha(\hat{t})d\hat{t}$, so it can be taken arbitrarily as we wish. Similarly, $e^{B(y)}$ is also an extra degree of freedom which can be taken arbitrarily as we wish. So we will take $B(y)$ properly in the metric (2.2) later.

The metric (2.2) contains the time-dependent scale factor, $\beta^2(\hat{t})$, for the internal dimensions, which means that the internal dimensions are basically allowed to evolve with time and this is one of the main points of our discussion distinguished from the usual higher-dimensional theories in which the volume of the internal space is fixed by $\beta^2(\hat{t}) = 1$ from the beginning. Since the metric of the internal space changes with time, we may have to allow the time-dependence of the other fields as well. We introduce an ansatz for the dilaton as

$$e^{\Phi(y,\hat{t})} = g_s \gamma(\hat{t}) e^{\Phi_S(y)} , \quad (2.4)$$

where g_s is the string constant. Similarly, the ansatz for the R-R four-form $A_{(4)}$ and the three-form $G_{(3)}$ are given respectively by

$$A_{(4)} = \sigma(\hat{t}) \xi(y) \sqrt{-\hat{g}_4} d\hat{t} \wedge dx^1 \wedge dx^2 \wedge dx^3 , \quad (2.5)$$

where \hat{g}_4 is the determinant of $\hat{g}_{\mu\nu}$ and therefore $\sqrt{-\hat{g}_4} = a^3(\hat{t})$, and

$$F_{(3)} = \eta(\hat{t}) \mathcal{F}_{(3)}(y) , \quad H_{(3)} = \eta(\hat{t}) \mathcal{H}_{(3)}(y) \quad \rightarrow \quad G_{(3)} = \eta(\hat{t}) \mathcal{G}_{(3)}(y, \hat{t}) , \quad (2.6)$$

where $\mathcal{G}_{(3)}(y, \hat{t}) \equiv \mathcal{F}_{(3)} - i \frac{\text{Im}\tau}{\gamma} \mathcal{H}_{(3)}$ with $\text{Im}\tau \equiv (g_s e^{\Phi_S})^{-1}$.

Upon reduction (2.2), and taking $B(y) = \Phi_S(y) - A(y)$, one finds that (2.1) reduces to

$$\begin{aligned} I_{\text{IIB}} = & \frac{1}{2\kappa_{10}^2 g_s^2} \left(\int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^4 \beta^4}{\gamma^2} \right) \left(\int d^6 y \sqrt{h_6} (\mathcal{R}_6(h_{mn}) - 2\mathcal{H}) \right) \\ & + \frac{1}{2\kappa_{10}^2 g_s^2} \left(\int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\sigma^2 \beta^4}{\alpha^4} \right) \left(\int d^6 y \sqrt{h_6} \frac{g_s^2}{2} e^{2\Phi_S - 4A} (\partial\xi)^2 \right) \\ & - \frac{1}{2\kappa_{10}^2 g_s^2} \left(\int d^4 \hat{x} \sqrt{-\hat{g}_4} \alpha^4 \eta^2 \right) \left(\int d^6 y \sqrt{h_6} \frac{g_s^2}{3!} e^{2A} \mathcal{G}_{mnp}^+ \bar{\mathcal{G}}^{+mnp} \right) \\ & + \frac{1}{2\kappa_{10}^2 g_s^2} \left(\int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^2 \beta^6}{\gamma^2} \mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma) \right) \left(\int d^6 y \sqrt{h_6} e^{\Phi_S - 2A} \right) \\ & + \text{topological terms} , \end{aligned} \quad (2.7)$$

where $\mathcal{H} \equiv \frac{1}{2}(\partial\Phi_S)^2 - (\partial\Phi_S)(\partial A) + (\partial A)^2$ and \mathcal{G}_{mnp}^+ represents the IASD piece of the \mathcal{G}_{mnp} , $\mathcal{G}_{(3)}^+ = \mathcal{G}_{(3)}^{\text{IASD}}$. Also, $\mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma)$ and topological terms are given respectively as follows.

First, $\mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma)$ represents

$$\mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma) = \mathcal{R}_4(\hat{g}_{\mu\nu}) + 6\frac{\ddot{\alpha}}{\alpha} + 12\frac{\ddot{\beta}}{\beta} + 30\left(\frac{\dot{\beta}}{\beta}\right)^2 + 18\frac{\dot{\alpha}}{\alpha}\frac{\dot{\alpha}}{\alpha} + 36\frac{\dot{\alpha}}{\alpha}\frac{\dot{\beta}}{\beta} + 24\frac{\dot{\alpha}}{\alpha}\frac{\dot{\beta}}{\beta} - 4\left(\frac{\dot{\gamma}}{\gamma}\right)^2, \quad (2.8)$$

where the "dot" denotes the derivative with respect to \hat{t} and $\mathcal{R}_4(\hat{g}_{\mu\nu})$ is the usual Ricci-scalar of the 4D metric (2.3):

$$\mathcal{R}_4(\hat{g}_{\mu\nu}) = 6\left(\frac{\ddot{\alpha}}{\alpha} + \left(\frac{\dot{\alpha}}{\alpha}\right)^2\right). \quad (2.9)$$

The above $\mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma)$ reduces to $\mathcal{R}_4(\hat{g}_{\mu\nu})$ in the time-independent limit $\alpha(\hat{t}) = \beta(\hat{t}) = \gamma(\hat{t}) = 1$. Also one can show that (2.8) can be rewritten as

$$\begin{aligned} \sqrt{-\hat{g}_4}\frac{\alpha^2\beta^6}{\gamma^2}\mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma) &= \frac{d}{d\hat{t}}\left[a^3\frac{\alpha^2\beta^6}{\gamma^2}\left(6\left(\frac{\dot{\alpha}}{\alpha} + \frac{\dot{\alpha}}{\alpha}\right) + 12\frac{\dot{\beta}}{\beta}\right)\right] \\ &+ a^3\frac{\alpha^2\beta^6}{\gamma^2}\left(-6\left(\frac{\dot{\alpha}}{\alpha} + \frac{\dot{\alpha}}{\alpha}\right)^2 + 12\left(\frac{\dot{\alpha}}{\alpha} + \frac{\dot{\alpha}}{\alpha}\right)\left(\frac{\dot{\gamma}}{\gamma} - 3\frac{\dot{\beta}}{\beta}\right) - 4\left(\frac{\dot{\gamma}}{\gamma} - 3\frac{\dot{\beta}}{\beta}\right)^2 + 6\left(\frac{\dot{\beta}}{\beta}\right)^2\right). \end{aligned} \quad (2.10)$$

The topological terms, on the other hand, are given by

$$\begin{aligned} \text{topological terms} &= \frac{i}{4\kappa_{10}^2}\left(\int d^4\hat{x}\sqrt{-\hat{g}_4}\alpha^4\eta^2\right)\left(\frac{1}{\text{Im}\tau}\int\frac{e^{2A-\Phi_s}}{g_s}\mathcal{G}_{(3)}\wedge\bar{\mathcal{G}}_{(3)}\right) \\ &- \frac{i}{4\kappa_{10}^2}\left(\int d^4\hat{x}\sqrt{-\hat{g}_4}\sigma\gamma\eta^2\right)\left(\frac{1}{\text{Im}\tau}\int\xi\mathcal{G}_{(3)}\wedge\bar{\mathcal{G}}_{(3)}\right), \end{aligned} \quad (2.11)$$

where $(\text{Im}\tau)^{-1} \equiv g_s e^{\Phi_s}$. In (2.11), the second term is just the Chern-Simons term $\int e^{\Phi}A_{(4)}\wedge G_{(3)}\wedge\bar{G}_{(3)}$. But the first term comes from the $G_{(3)}\cdot\bar{G}_{(3)}$ term of the action (2.1). Using the identity

$$\mathcal{G}_{(3)}\wedge*_6\bar{\mathcal{G}}_{(3)} = -i\mathcal{G}_{(3)}\wedge\bar{\mathcal{G}}_{(3)} + 2i\mathcal{G}_{(3)}^+\wedge\bar{\mathcal{G}}_{(3)}^+, \quad (2.12)$$

together with $*_6\mathcal{G}_{(3)}^+ = -i\mathcal{G}_{(3)}^+$, one can show that the $G_{(3)}\cdot\bar{G}_{(3)}$ term in (2.1) can be decomposed as

$$\begin{aligned} -\frac{1}{24\kappa_{10}^2}\int d^{10}x\sqrt{-G_{10}}G_{(3)}\cdot\bar{G}_{(3)} &= \left(\int d^4\hat{x}\sqrt{-\hat{g}_4}\alpha^4\eta^2\right)\left(\frac{i}{4\kappa_{10}^2}\frac{1}{\text{Im}\tau}\int\frac{e^{2A-\Phi_s}}{g_s}\mathcal{G}_{(3)}\wedge\bar{\mathcal{G}}_{(3)}\right) \\ &- \left(\int d^4\hat{x}\sqrt{-\hat{g}_4}\alpha^4\eta^2\right)\left(\frac{1}{12\kappa_{10}^2}\int d^6y\sqrt{h_6}e^{2A}\mathcal{G}_{mnp}^+\bar{\mathcal{G}}^{+mnp}\right), \end{aligned} \quad (2.13)$$

and these two terms become, respectively, the first term of (2.11) and the third term of (2.7).

III. Brane action

In addition to the action I_{IIB} , we also have local terms

$$I_{\text{brane}} = - \int d^4 \hat{x} \sqrt{-\det(G_{\mu\nu})} T(\Phi) + \mu(\Phi) \int A_{(4)} , \quad (3.1)$$

where $G_{\mu\nu}$ is a pullback of the target space metric G_{MN} to the 4D brane world. In (3.1), $T(\phi)$ represents the tension of the $D3$ -brane; it is given by $T(\Phi) = T_0 e^{-\Phi}$ at the tree level, but $T(\Phi) = T_0 e^{-\Phi} + \rho_{\text{vac}}(\Phi)$ at the quantum level, where $\rho_{\text{vac}}(\Phi)$ represents quantum correction terms; $\rho_{\text{vac}}(\Phi) = \sum_{n=0}^{\infty} T_{n+1} e^{n\Phi}$ (see, for instance, Ref. [7]). So $T(\Phi)$ becomes

$$T(\Phi) = T_0 e^{-\Phi} \Gamma_{NS}(\Phi) , \quad (3.2)$$

where

$$\Gamma_{NS}(\Phi) = 1 + \sum_{n=1}^{\infty} \hat{T}_n e^{n\Phi} , \quad (\hat{T} \equiv \frac{T_n}{T_0}) . \quad (3.3)$$

Similarly, $\mu(\Phi)$ is given by $\mu(\Phi) = \mu_0$ at the tree level, but it turns into $\mu(\Phi) = \mu_0 + \delta\mu(\Phi)$ at the quantum level where $\delta\mu(\Phi)$ is given by $\delta\mu(\Phi) = \sum_{n=1}^{\infty} \mu_n e^{n\Phi}$. So $\mu(\Phi)$ becomes

$$\mu(\Phi) = \mu_0 \Gamma_R(\Phi) , \quad (3.4)$$

where

$$\Gamma_R(\Phi) = 1 + \sum_{n=1}^{\infty} \hat{\mu}_n e^{n\Phi} , \quad (\hat{\mu}_n \equiv \frac{\mu_n}{\mu_0}) . \quad (3.5)$$

Using (2.5) together with (3.2) and (3.4), one finds that (3.1) reduces to

$$I_{\text{brane}} = \int d^4 \hat{x} \sqrt{-\hat{g}_4} \int d^6 y \sqrt{h_6} \left[-\frac{T_0}{g_s} \chi^{1/2}(y) \frac{\alpha^4}{\gamma} \Gamma_{NS} + \mu_0 \xi(y) \sigma \Gamma_R \right] \delta^6(y) , \quad (3.6)$$

where the 6D delta function is normalized by $\int d^6 y \sqrt{h_6} \delta^6(y) = 1$ and χ is defined by

$$\chi = e^{4A-2\Phi_S} . \quad (3.7)$$

In (3.6), the first term constitutes the NS-NS part of the action, while the second term is an R-R counterpart of the first term. For the BPS-branes ($T_0 = \mu_0$) these two terms cancel out at the tree level, which is related with the fact that $D3$ -brane potential defined by

$$\frac{1}{g_s} \Phi_- \equiv \frac{\chi^{1/2}}{g_s} - \xi \quad (3.8)$$

vanishes at the imaginary self-dual (ISD) backgrounds. Indeed, these two terms are expected to cancel out to all orders of perturbations when supersymmetry of the brane

region is unbroken. But in (3.6), such a cancellation cannot be achieved unless the time-dependent factor $\frac{\alpha^4}{\gamma}$ of the first term coincides with σ of the second term. So we choose $\alpha(\hat{t})$ as

$$\alpha^4 = \sigma\gamma , \quad (3.9)$$

so that the cancelation occurs for the BPS-branes.

IV. Equation of motion for $\xi(y)$ and 6D Einstein equation

4.1 Equation of motion for $\xi(y)$

Now we turn to the equations of motion. We first consider the equation of motion for $\xi(y)$. From (2.7) and (3.6), the 10D Lagrangian for $\xi(y)$ can be written as

$$\begin{aligned} 2\kappa_{10}^2 L_{\xi(y)} = & \frac{1}{2} \sqrt{-\hat{g}_4} \sqrt{h_6} \frac{\sigma^2 \beta^4}{\alpha^4} \chi^{-1} (\partial\xi)^2 + \frac{i}{12} \sqrt{-\hat{g}_4} \sqrt{h_6} \sigma \gamma \eta^2 \frac{\xi}{\text{Im}\tau} \mathcal{G}_{mnp} (*_6 \bar{\mathcal{G}})^{mnp} \\ & + 2\kappa_{10}^2 \sqrt{-\hat{g}_4} \sqrt{h_6} \mu_0 \sigma \Gamma_R \xi(y) \delta^6(y) , \end{aligned} \quad (4.1)$$

(where the second term comes from the topological term in (2.11).) and from this Lagrangian we obtain the equation of motion

$$\frac{1}{\sqrt{h_6}} \partial^m \left(\sqrt{h_6} \chi^{-1} h_{mn} (\partial^n \xi) \right) = \frac{i}{12} \frac{\alpha^4 \gamma}{\beta^4 \sigma} \eta^2 \frac{1}{\text{Im}\tau} \mathcal{G}_{mnp} (*_6 \bar{\mathcal{G}})^{mnp} + 2\kappa_{10}^2 \mu_0 \frac{\alpha^4}{\beta^4} \frac{\Gamma_R}{\sigma} \delta^6(y) . \quad (4.2)$$

(4.2) differs from the corresponding equation of the time-independent theory in [3]. The left hand side is independent of \hat{t} as in [3]. But each term on the right hand side contains extra factors $\frac{\alpha^4 \gamma}{\beta^4 \sigma} \eta^2$ and $\frac{\alpha^4 \Gamma_R}{\beta^4 \sigma}$ respectively, which are functions of \hat{t} . So in order that the equality holds we must require that the functions $\frac{\alpha^4 \gamma}{\beta^4 \sigma} \eta^2$ and $\frac{\alpha^4 \Gamma_R}{\beta^4 \sigma}$ must be constants. We set

$$\frac{\alpha^4 \gamma}{\beta^4 \sigma} \eta^2 = 1 , \quad (4.3)$$

and similarly

$$\frac{\alpha^4}{\beta^4} \frac{\Gamma_R}{\sigma} = 1 , \quad (4.4)$$

where Γ_R represents the values of $\Gamma_R(\Phi(y, \hat{t}))$ at $y = 0$ by the Dirac delta $\delta^6(y)$, so it is only a function of \hat{t} . By (4.3) and (4.4), (4.2) reduces to the time-independent equation

$$\nabla^2 \xi = \frac{i}{12 \text{Im}\tau} \chi \mathcal{G}_{mnp} (*_6 \bar{\mathcal{G}})^{mnp} + 2\chi^{-1/2} (\partial\chi^{1/2}) (\partial\xi) + 2\kappa_{10}^2 \mu_0 \chi \delta^6(y) . \quad (4.5)$$

Equation (4.5) corresponds to the Bianchi identity $d\tilde{F}_{(5)} = H_{(3)} \wedge F_{(3)} + 2\kappa_{10}^2 \mu_0 \delta^6(y)$ of the Einstein frame and it coincides with the corresponding equation of the time-independent theory in [3] (see Eq. (7.7) of Ref. [3]).

4.2 6D Einstein equations

Now we consider the 6D Einstein equations which can be obtained from (2.7). The type IIB action (2.7) can be simplified as follows. From (3.9) and (4.3) one finds that

$$\eta = \frac{\beta^2}{\gamma} , \quad (4.6)$$

and using (3.9) and (4.6) one obtains a time-independent 6D action from (2.7) :

$$I_{\text{IIB}} / \left(\int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^4 \beta^4}{\gamma^2} \right) = \frac{1}{2\kappa_{10}^2 g_s^2} \int d^6 y \sqrt{h_6} \left(\mathcal{R}_6(h_{mn}) - \mathcal{L}_F + c \chi^{-1/2} \right) + \hat{I}_{\text{topological}} , \quad (4.7)$$

where \mathcal{L}_F and c are given by

$$\mathcal{L}_F = 2\mathcal{H} - \frac{g_s^2}{2} \chi^{-1} (\partial\xi)^2 + \frac{g_s^2}{3!} e^{2A} \mathcal{G}_{mnp}^+ \bar{\mathcal{G}}^{+mnp} , \quad (4.8)$$

and

$$c = \frac{\int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^2 \beta^6}{\gamma^2} \mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma)}{\int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^4 \beta^4}{\gamma^2}} . \quad (4.9)$$

Also the topological term $\hat{I}_{\text{topological}}$ is defined by

$$\hat{I}_{\text{topological}} = \frac{i}{4\kappa_{10}^2 \text{Im}\tau} \int \left(\frac{\chi^{1/2}}{g_s} - \xi \right) \mathcal{G}_{(3)} \wedge \bar{\mathcal{G}}_{(3)} , \quad (4.10)$$

but this term does not contribute to the 6D Einstein equations because it does not contain the 6D metric h^{mn} .

The action (4.7) coincides with the corresponding action in Ref. [3] only except that β is replaced by a new constant c . (see Eq. (3.7) of Ref. [3]). Namely, (4.7) defines a time-independent theory in which the field equations are given by those of [3]. So once we define 6D effective action as in (4.7), what we are considering is a time-independent field theory because (4.7) essentially consists of the time-independent fields defined on the 6D internal space \mathcal{M}_6 . The only thing that requires a little more explanation is that what happens when we go up to the quantum level. We see that quantum corrections of (4.7) might be expressed in a g_s -expansion where the expansion parameter is given by the dilaton $e^{\Phi(y, \hat{t})}$ in (2.4). So at the quantum level (4.7) begins to contain $\gamma(\hat{t})$ through

the dilaton $e^{\Phi(y,\hat{t})}$ (Indeed the three-form $\mathcal{G}_{(3)}$ contains $\gamma(\hat{t})$ even at the leading order), and it changes (4.7) into a time-dependent action in that case.

But still, we can remain in the time-independent theory if we use an approximation in which $\gamma(\hat{t})$ of the g_s -expansion is replaced by its present value $\gamma(\hat{t}_0)$ which is virtually equal to one (see Sec. X). This is a good approximation because $\gamma(\hat{t})$ is effectively constant in a short time interval (of the integration $\int d\hat{t}$) of the present stage of our universe. Also, this is a natural approximation because in this approximation the theory remains time-independent regardless of whether we are at the tree level or quantum level. In this paper we will use this approximation in which the expansion parameter of the g_s -perturbation is given by the usual $g_s e^{\Phi_s}$ (i.e. we will set $\gamma(\hat{t}) = 1$) in the theories described by the time-independent actions such as (4.7). Indeed it is shown in Sec. X that $\gamma(t) = 1$ is the most natural solution even when the internal dimensions evolve with time (see Eqs. (10.15) and (10.49)). So if we take this as our solution, then we do not even need to use the approximation described above. The theory will always remain time-independent regardless of whether we are at the tree level or quantum level.

Varying (4.7) with respect to δh^{mn} , one obtains

$$\mathcal{R}_{mn} - \frac{1}{2}h_{mn}\mathcal{R}_6 - \frac{1}{2}T_{mn} - \frac{c}{2}\chi^{-1/2}h_{mn} = 0 , \quad (4.11)$$

where the energy-momentum tensor T_{mn} is defined by

$$T_{mn} = \frac{2}{\sqrt{h_6}} \frac{\delta I_F}{\delta h^{mn}} , \quad (I_F \equiv \int d^6y \sqrt{h_6} \mathcal{L}_F) . \quad (4.12)$$

In (4.11), we do not take $\mathcal{R}_{mn} = \mathcal{R}_6 = 0$, though they vanish at the classical level. In our perturbation scheme (see Eq.(2.13) of Ref. [8], for instance) the metric acquires the correction terms

$$h_{mn} = h_{mn}^{(0)} + h_{mn}^{(1)} + h_{mn}^{(2)} + \cdots = h_{mn}^{(0)} + \delta_Q h_{mn} \quad (4.13)$$

at the quantum level. So, $\mathcal{R}_{mn}(h_{mn})$ and $\mathcal{R}_6(h_{mn})$ do not vanish off-shell, though we have $\mathcal{R}_{mn}(h_{mn}^{(0)}) = \mathcal{R}_6(h_{mn}^{(0)}) = 0$.

V. 4D effective action

5.1 Total action I_{total}

In this section we consider the 4D effective action which will be used in Sec. VI to introduce λ , and also used in Sec. IX to obtain 4D equations of motion for β_Q , β and

A. To find 4D effective action, we first consider the relations

$$\frac{\alpha^4 \beta^4}{\gamma^2} = \frac{\alpha^4}{\beta^4} \Gamma_R^2, \quad \frac{\alpha^2 \beta^6}{\gamma^2} = \frac{\alpha^2}{\beta^2} \Gamma_R^2, \quad (5.1)$$

which can be obtained from (3.9) and (4.4). Using (5.1) (and also using (3.9) and (4.6)) one can rewrite I_{IIB} in (2.7) as

$$I_{\text{IIB}} = \frac{1}{2\kappa^2} \int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^2}{\beta^2} \Gamma_R^2 \mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma) + \int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^4}{\beta^4} \Gamma_R^2 (\hat{I}_{\text{bulk}} + \hat{I}_{\text{topological}}), \quad (5.2)$$

where $2\kappa^2 \equiv 2\kappa_{10}^2 g_s^2 / (\int d^6 y \sqrt{h_6} \chi^{-1/2})$ and \hat{I}_{bulk} is given by

$$\hat{I}_{\text{bulk}} = \frac{1}{2\kappa_{10}^2 g_s^2} \int d^6 y \sqrt{h_6} (\mathcal{R}_6(h_{mn}) - \mathcal{L}_F). \quad (5.3)$$

Similarly, using (3.9) and (4.4) one can show that I_{brane} in (3.6) can be rewritten as

$$I_{\text{brane}} = \int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^4}{\beta^4} \Gamma_R^2 \hat{I}_{\text{brane}}, \quad (5.4)$$

where \hat{I}_{brane} is the brane action density

$$\hat{I}_{\text{brane}} \equiv \int d^6 y \sqrt{h_6} \left(-\frac{T_0}{g_s} \chi^{1/2}(y) l(\gamma(\hat{t})) + \mu_0 \xi(y) \right) \delta^6(y), \quad (5.5)$$

and in (5.5) $l(\gamma(\hat{t}))$ is defined by

$$l(\gamma(\hat{t})) \equiv \lim_{y \rightarrow 0} \frac{\Gamma_{NS}(\Phi)}{\Gamma_R(\Phi)} = 1 + g_s e^{\Phi_S(0)} (\hat{T}_1 - \hat{\mu}_1) \gamma(\hat{t}) + \dots. \quad (5.6)$$

$l(\gamma(\hat{t}))$ becomes $l(\gamma(\hat{t})) = 1$ when the branes are BPS ($\hat{T}_n = \hat{\mu}_n$). So in this case \hat{I}_{brane} in (5.5) takes the tree level form

$$\hat{I}_{\text{brane}}(\text{tree}) = \int d^6 y \sqrt{h_6} \left(-\frac{T_0}{g_s} \chi^{1/2}(y) + \mu_0 \xi(y) \right) \delta^6(y), \quad (5.7)$$

and it can be shown that this $\hat{I}_{\text{brane}}(\text{tree})$ vanishes by the field equations for $\chi(y)$ and $\xi(y)$ (see Ref. [6] or Sec. 8.2 of this paper). Indeed the integrand of \hat{I}_{brane} acts as a $D3$ -brane potential (see (3.8)) and it is known that it vanishes for $\mu_0 = T_0$ at the tree level. But once the brane supersymmetry is broken by the perturbations, \hat{I}_{brane} acquires nonvanishing correction terms coming from the quantum fluctuations and in this case \hat{I}_{brane} does not vanish anymore.

Now the total 4D effective action can be obtained by adding (5.4) to (5.2). We have

$$I_{\text{total}} = \frac{1}{2\kappa^2} \int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^2}{\beta^2} \Gamma_R^2 \mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma) + \int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^4}{\beta^4} \Gamma_R^2 \hat{I}_{\text{total}}, \quad (5.8)$$

where \hat{I}_{total} is defined by

$$\hat{I}_{\text{total}} \equiv \hat{I}_{\text{bulk}} + \hat{I}_{\text{brane}} + \hat{I}_{\text{topological}} , \quad (5.9)$$

while $\mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma)$ is given by (2.8) or (2.10). I_{total} in (5.8) will be identified as the 4D effective action in the next section and from this action we will obtain λ in Sec. VI and equations of motion in Sec. IX.

5.2 I_{total} as a 4D effective action

(5.8) contains the curvature scalar of the 4D spacetime metric $\hat{g}_{\mu\nu}$ (see (2.8)) and therefore it can be used as a 4D effective action containing gravity. However, the curvature term contained in (5.8) is not the standard Hilbert-Einstein action $\frac{1}{2\kappa^2} \int d^4x \sqrt{-g_4} \mathcal{R}_4$ of the gravity. So in order to obtain 4D action with the standard Hilbert-Einstein action we need some procedure given below.

In the first term of (5.8), $\mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma)$ was previously given by both (2.8) and (2.10). In this section we choose (2.10) to start our discussion. Using

$$\frac{\beta^3}{\gamma} = \frac{\Gamma_R}{\beta} , \quad (5.10)$$

which follows from (3.9) and (4.4), one can rewrite (2.10) in more convenient form as

$$\begin{aligned} \sqrt{-\hat{g}_4} \frac{\alpha^2}{\beta^2} \Gamma_R^2 \mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma) &= \frac{d}{d\hat{t}} \left(\frac{\mathcal{A}^3 \beta_Q}{\alpha} \left(6 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} + \frac{\dot{\beta}_Q}{\beta_Q} \right) + 12 \frac{\dot{\beta}}{\beta} \right) \right) \\ &+ \frac{\mathcal{A}^3 \beta_Q}{\alpha} \left(-6 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 + 2 \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + 6 \left(\frac{\dot{\beta}}{\beta} \right)^2 \right) , \end{aligned} \quad (5.11)$$

where \mathcal{A} and β_Q are defined, respectively, by

$$\mathcal{A} \equiv a\alpha \frac{\Gamma_R}{\beta} , \quad \beta_Q \equiv \frac{\beta}{\Gamma_R} . \quad (5.12)$$

Now we make a coordinate transformation $\hat{t} \rightarrow t$ defined by

$$dt = \frac{\alpha}{\beta_Q} d\hat{t} . \quad (5.13)$$

Then, using (5.11) (and also (5.13)) one can rewrite the first term of (5.8) as

$$\begin{aligned} \frac{1}{2\kappa^2} \int d^4\hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^2}{\beta^2} \Gamma_R^2 \mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma) &= \frac{1}{2\kappa^2} \int d^3\vec{x} \int dt \left[\frac{d}{dt} \left(\mathcal{A}^3 \left(6 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} + \frac{\dot{\beta}_Q}{\beta_Q} \right) + 12 \frac{\dot{\beta}}{\beta} \right) \right) \right. \\ &\left. + \mathcal{A}^3 \left(-6 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 + 2 \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + 6 \left(\frac{\dot{\beta}}{\beta} \right)^2 \right) \right] , \end{aligned} \quad (5.14)$$

where the "dot" now denotes the derivative with respect to t .

(5.14) is the 4D effective action for the curvature defined on the 4D sector (t, \vec{x}) of the 10D spacetime whose metric is now given by (see (2.2), (5.12), (5.13) and (5.10))

$$ds_{10}^2 = \frac{\gamma^2(t)}{\beta^6(t)} e^{A(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + \beta^2(t) e^{\Phi_S(y) - A(y)} h_{mn}(y) dy^m dy^n, \quad (5.15)$$

where the 4D metric $g_{\mu\nu}(x) dx^\mu dx^\nu$ is defined by

$$g_{\mu\nu}(x) dx^\mu dx^\nu = -dt^2 + \mathcal{A}^2(t) d\vec{x}_3^2. \quad (5.16)$$

Indeed (5.14) can be recast into

$$\frac{1}{2\kappa^2} \int d^4\hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^2}{\beta^2} \Gamma_R^2 \mathcal{R}_4^{(\text{eff})}(\hat{g}_{\mu\nu}, \alpha, \beta, \gamma) = \frac{1}{2\kappa^2} \int d^3\vec{x} \int dt \sqrt{-g_4} \mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta) \quad (5.17)$$

with $\mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta)$ defined by

$$\mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta) = \mathcal{R}_4(g_{\mu\nu}) + 6 \frac{d}{dt} \left(\frac{\dot{\beta}_Q}{\beta_Q} \right) + 12 \frac{d}{dt} \left(\frac{\dot{\beta}}{\beta} \right) + 2 \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + 6 \left(\frac{\dot{\beta}}{\beta} \right)^2 + 18 \frac{\dot{\mathcal{A}}}{\mathcal{A}} \frac{\dot{\beta}_Q}{\beta_Q} + 36 \frac{\dot{\mathcal{A}}}{\mathcal{A}} \frac{\dot{\beta}}{\beta}, \quad (5.18)$$

where $\mathcal{R}_4(g_{\mu\nu})/\sqrt{-g_4}$ are the Ricci-Scalar/determinant of the 4D metric (5.16), respectively. So we have $\mathcal{R}_4(g_{\mu\nu}) = 6 \left(\frac{\ddot{\mathcal{A}}}{\mathcal{A}} + \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 \right)$ and $\sqrt{-g_4} = \mathcal{A}^3$. $\mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta)$ in (5.18) reduces to $\mathcal{R}_4(g_{\mu\nu})$ in the time-independent limit $\dot{\beta}_Q = \dot{\beta} = 0$. So in the coordinate system where the metric is given by (5.15), (5.17) becomes the standard 4D Hilbert-Einstein action of the metric (5.16) in the limit $\dot{\beta}_Q = \dot{\beta} = 0$. Now using (5.17) (and also using (5.12) and (5.13)) one can show that (5.8) finally takes the form

$$I_{\text{total}} = \frac{1}{2\kappa^2} \int d^3\vec{x} \int dt \sqrt{-g_4} \mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta) + \int d^3\vec{x} \int dt \sqrt{-g_4} \frac{\beta_Q^2}{\beta^2} \hat{I}_{\text{total}}. \quad (5.19)$$

where the first terms is given by both (5.14) and (5.18).

VI. 4D cosmological constant

I_{total} in (5.19) can be rewritten as

$$I_{\text{total}} = \frac{1}{2\kappa^2} \int d^3\vec{x} \int dt \sqrt{-g_4} \left(\mathcal{R}_4(g_{\mu\nu}) - 2\lambda \right), \quad (6.1)$$

where λ is the cosmological constant defined by

$$\lambda = -\kappa^2 \left(\frac{\beta_Q}{\beta} \right)^2 \hat{I}_{\text{total}} - \frac{1}{2} \Delta \mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta), \quad (6.2)$$

where $\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ represents the extra terms in (5.18):

$$\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta) \equiv 6\frac{d}{dt}\left(\frac{\dot{\beta}_Q}{\beta_Q}\right) + 12\frac{d}{dt}\left(\frac{\dot{\beta}}{\beta}\right) + 2\left(\frac{\dot{\beta}_Q}{\beta_Q}\right)^2 + 6\left(\frac{\dot{\beta}}{\beta}\right)^2 + 18\frac{\dot{\mathcal{A}}}{\mathcal{A}}\frac{\dot{\beta}_Q}{\beta_Q} + 36\frac{\dot{\mathcal{A}}}{\mathcal{A}}\frac{\dot{\beta}}{\beta} . \quad (6.3)$$

In (6.2), β and β_Q (and also $\dot{\beta}$ and $\dot{\beta}_Q$) all represent their present values because (6.2) is the cosmological constant of the present universe. (6.2) suggests that not only are the (quantum corrections of) \hat{I}_{total} the contributions to λ . The time evolution (i.e. nonzero $\dot{\beta}_Q$ and $\dot{\beta}$) of the internal dimensions also contributes to λ in the present case. But in the limit $\beta_Q = \beta = 1$, (6.2) reduces to the equation

$$\lambda = -\kappa^2 \hat{I}_{\text{total}} , \quad (6.4)$$

which coincides with Eq. (3.15) of Ref. [3] if we ignore $\hat{I}_{\text{topological}}$ in \hat{I}_{total} .²

Going back to Sec. 4.2 we see that \mathcal{L}_F in (4.8) includes both kinetic and potential terms, where the former takes the form $K = h^{mn}K_{mn}$ with K_{mn} given by $K_{mn} = \sum_{A,B} F_{AB}(\phi_C)\partial_m\phi_A\partial_n\phi_B$, where ϕ_A represent the 6D scalars such as Φ_S , A and ξ etc. So in the kinetic part of \mathcal{L}_F , K_{mn} does not involve any metric h^{mn} , but the potential part V of \mathcal{L}_F (i.e. $\mathcal{G}_{mnp}^+\bar{\mathcal{G}}^{+mnp}$ term in the case of (4.8)) includes h^{mn} : $V = V(\phi_A, h^{mn})$, where V is related to the scalar potential $\mathcal{V}_{\text{scalar}}$ of the $N = 1$ supergravity by the equation

$$\mathcal{V}_{\text{scalar}} = \frac{1}{2\kappa_{10}^2 g_s^2} \int d^6y \sqrt{h_6} V . \quad (6.5)$$

(See Eq. (3.17) (and also the sentences below it) of the Ref. [3].) In any case, \mathcal{L}_F in (4.8) generally takes the form

$$\mathcal{L}_F = K - V , \quad (K = h^{mn}K_{mn}) , \quad (6.6)$$

and if we substitute (4.12) (with \mathcal{L}_F given by (6.6)) into (4.11), we obtain (after contracting the indices m and n)

$$\mathcal{R}_6 - \mathcal{L}_F - \frac{1}{2}(\mathcal{N} - 1)V + \frac{3}{2}c\chi^{-1/2} = 0 , \quad (6.7)$$

where \mathcal{N} is defined by $\mathcal{N} \equiv h^{mn}\frac{\partial}{\partial h^{mn}}$ and c (which was defined by (4.9)) is now given by

$$c = \frac{\int d^3\vec{x} \int dt \sqrt{-g_4} \mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta)}{\int d^3\vec{x} \int dt \sqrt{-g_4} \frac{\beta_Q^2}{\beta^2}} . \quad (6.8)$$

²Indeed (3.15) of Ref. [3] must contain $\hat{I}_{\text{topological}}$ as in (6.4) of this paper. But the omission of $\hat{I}_{\text{topological}}$ in [3] will not change the story of Ref. [3] at all because this $\hat{I}_{\text{topological}}$ is always gauged away together with \hat{I}_{bulk} and \hat{I}_{brane} in \hat{I}_{total} anyway. (See Sec. 1 of Ref. [3] or see Sec. 7.2 of this paper.)

(See (5.1) and (5.17) together with (5.12) and (5.13).)

(6.8) can be rewritten as

$$c = \frac{\beta^2}{\beta_Q^2} \mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta) , \quad (6.9)$$

because in (6.8) $g_{\mu\nu}$, β and β_Q (and their derivatives) all represent their present values of our universe and therefore $\mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta)$ and $\frac{\beta_Q^2}{\beta^2}$ in (6.8) are effectively constants in a short time interval of the present stage of our universe. Indeed, $\mathcal{R}_4(g_{\mu\nu})$ in $\mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta)$ is generally given by $\mathcal{R}_4(g_{\mu\nu}) = 4\lambda$ for the maximally symmetric spacetime (which means that we put $\mathcal{A}(t) = e^{\sqrt{\frac{\lambda}{3}}t}$ in the metric (5.16)) and similarly $\frac{\dot{\mathcal{A}}}{\mathcal{A}}$, $\frac{\dot{\beta}_Q}{\beta_Q}$ and $\frac{\dot{\beta}}{\beta}$ will be taken as constants in the 4D equations of motion in Sec.X. So (6.9) becomes

$$c = \frac{\beta^2}{\beta_Q^2} \left(4\lambda + \Delta \mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta) \right) \quad (6.10)$$

for the metric (5.16) with $\mathcal{A}(t)$ given by $\mathcal{A}(t) = e^{\sqrt{\frac{\lambda}{3}}t}$.

Now we integrate (6.7) and use (5.3) and (6.10) to obtain

$$\hat{I}_{\text{bulk}} = -\frac{3}{4\kappa^2} \frac{\beta^2}{\beta_Q^2} \left(4\lambda + \Delta \mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta) \right) + \frac{1}{4\kappa_{10}^2 g_s^2} \int d^6 y \sqrt{h_6} (\mathcal{N} - 1) V . \quad (6.11)$$

Also, substituting (6.11) into (6.2) we finally get

$$\lambda = \frac{\beta_Q^2}{\beta^2} \left(\frac{\kappa^2}{8\kappa_{10}^2 g_s^2} \int d^6 y \sqrt{h_6} (\mathcal{N} - 1) V + \frac{\kappa^2}{2} (\hat{I}_{\text{brane}} + \hat{I}_{\text{topological}}) \right) - \frac{1}{8} \Delta \mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta) , \quad (6.12)$$

which is the generalization of Eq. (3.20) of Ref. [3].³ Note that we have not set $\mathcal{R}_{mn} = \mathcal{R}_6 = 0$ in the whole procedure of obtaining (6.12) because these quantities do not vanish off-shell. But λ in (6.12) does not include $\mathcal{R}_6(h_{mn})$ or $\mathcal{R}_{mn}(h_{mn})$. They cancel themselves out during the process of obtaining (6.12) and as a result λ becomes independent of $\mathcal{R}_6(h_{mn})$.

VII. Self-tuning mechanism

7.1 Self-tuning equation for λ

³Again, Eq.(3.20) of Ref. [3] must include $\hat{I}_{\text{topological}}$. That is, the term $\frac{\kappa^2}{2} \hat{I}_{\text{brane}}$ must be replaced by $\frac{\kappa^2}{2} (\hat{I}_{\text{brane}} + \hat{I}_{\text{topological}})$ in that equation.

In this section we use the self-tuning mechanism in [3] to obtain a self-tuning equation for λ of our present model in which the internal dimensions are allowed to evolve with time. We first substitute \mathcal{L}_F in (6.7) into (4.12) to get

$$T_{mn} = 2(\mathcal{R}_{mn} - \frac{1}{2}h_{mn}\mathcal{R}_6) + \frac{1}{2}h_{mn}(\mathcal{N} - 1)V - \frac{\partial}{\partial h^{mn}}(\mathcal{N} - 1)V - \frac{3}{2}c\chi^{-1/2}h_{mn} . \quad (7.1)$$

Next, substitute (7.1) into (4.11) and contract m and n . Then we obtain a constraint equation for c (or a self-tuning equation for λ as we will see soon)

$$c = -\frac{1}{3}\chi^{1/2}(\mathcal{N} - 1)(\mathcal{N} - 3)V , \quad (7.2)$$

which demands that c must vanish if the potential density V has a certain tensor structure described below.

The constraint equation (7.2) can be generalized by using the procedure presented in [3]. The most general form of the constraint equation for c is

$$c = \frac{1}{6}\chi^{1/2}(\mathcal{N} - 1)(\mathcal{N} - 3)(1 - 3b_0\Pi(\mathcal{N}))V , \quad (7.3)$$

where b_0 is a constant and $\Pi(\mathcal{N})$ is an operator of the form

$$\Pi(\mathcal{N}) = \sum_k c_k(\mathcal{N} - n_1) \cdots (\mathcal{N} - n_k) , \quad (7.4)$$

where n_k are integers. (7.3) requires that c must vanish at least if V belongs to V_n ($V \in V_n$) with $n = 1$ or 3 , where V_n represents a class of potential densities satisfying

$$\mathcal{N}V_n = nV_n . \quad (7.5)$$

Indeed in the usual flux compactifications where the three-forms are used to stabilize the complex structure moduli, V basically belongs to V_3 (see [3]) and therefore c must vanish by (7.3): $c = 0$. This constraint equation $c = 0$ becomes a self-tuning equation $\lambda = 0$ in the theories where $\beta(t)$ and $\beta_Q(t)$ are given by $\beta(t) = \beta_Q(t) = 1$ because in that case $\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ vanishes and therefore λ becomes $\lambda = \frac{\epsilon}{4}$ in the equation (6.10).

In our present model we basically consider the flux compactifications where the complex structure moduli are stabilized by the three-form flux $G_{(3)}$. But still we do not exclude the conventional compactifications in which the three-form fluxes are turned off. In these latter cases c also vanishes by (7.3) because in these cases V itself vanishes by $\mathcal{G}_{(3)} = 0$ (note that V is given by $V \propto \mathcal{G}_{(3)}^+ \cdot \bar{\mathcal{G}}_{(3)}^+$) and therefore c trivially vanishes by (7.3). Hence in both cases c vanishes by (7.3) and we obtain

$$\lambda = -\frac{1}{4}\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta) \quad (7.6)$$

from (6.10). In (7.6), $\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$, and therefore λ vanishes in the time-independent compactifications with $\beta_Q(t) = \beta(t) = 1$. But once we allow the internal dimensions to evolve with time, λ can acquire nonzero values from the nonvanishing $\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ as one can see from (7.6).

7.2 \hat{I}_{brane} and $\hat{I}_{\text{topological}}$ in λ

Now we have two independent equations for λ . We have (6.12), and also we have (7.6) which is equivalent to $c = 0$. Since (7.6) follows from the constraint $c = 0$, it acts as a constraint equation for λ and in the case $\beta(t) = \beta_Q(t) = 1$ it really becomes a self-tuning equation $\lambda = 0$ as mentioned above. But in the case of nonvanishing $\dot{\beta}(t)$ and $\dot{\beta}_Q(t)$ it takes the general form (7.6) because in that case $\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ is not a vanishing quantity anymore.

The other equation (6.12), on the other hand, tells about the constituents of λ . For the potential density $V \in V_n$, it reduces to

$$\lambda = \frac{\beta_Q^2}{\beta^2} \left(\frac{(n-1)}{4} \kappa^2 \mathcal{V}_{\text{scalar}} + \frac{\kappa^2}{2} (\hat{I}_{\text{brane}} + \hat{I}_{\text{topological}}) \right) - \frac{1}{8} \Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta) , \quad (7.7)$$

which shows that λ is composed of three parts: i.e. a scalar potential $\mathcal{V}_{\text{scalar}}$, brane plus topological action density $\hat{I}_{\text{brane}} + \hat{I}_{\text{topological}}$ and finally $\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ coming from the nonvanishing $\dot{\beta}_Q$ and $\dot{\beta}$. Among these quantities, $\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ term vanishes in the time-independent theories in which $\beta(t)$ and $\beta_Q(t)$ are given by $\beta(t) = \beta_Q(t) = 1$. For $n = 3$, (7.7) becomes

$$\lambda = \frac{\kappa^2}{2} \frac{\beta_Q^2}{\beta^2} \left(\mathcal{V}_{\text{scalar}} + \hat{I}_{\text{brane}} + \hat{I}_{\text{topological}} \right) - \frac{1}{8} \Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta) , \quad (7.8)$$

and this reduces to Eq. (3.44) of Ref. [3] in the limit $\beta(t) = \beta_Q(t) = 1$.⁴

Among the constituents in (7.8), \hat{I}_{brane} can be decomposed further into three parts. We have

$$\hat{I}_{\text{brane}} = \left(\hat{I}_{\text{brane}}^{(NS)}(\text{tree}) + \hat{I}_{\text{brane}}^{(R)}(\text{tree}) \right) + \left(\delta_Q \hat{I}_{\text{brane}}^{(NS)} + \delta_Q \hat{I}_{\text{brane}}^{(R)} \right) - \mathcal{E}_{\text{SB}} . \quad (7.9)$$

(See Sec. 3.5 of Ref. [3] for this.) In (7.9), $\hat{I}_{\text{brane}}^{(NS)}(\text{tree})$ and $\hat{I}_{\text{brane}}^{(R)}(\text{tree})$ are the (NS-NS and R-R parts of the) tree-level actions and they always cancel out by field equations for the BPS $D3$ -branes. (We will show this briefly in Sec. VIII.) The next terms $\delta_Q \hat{I}_{\text{brane}}^{(NS)}$

⁴As in (3.15) and (3.20), (3.44) of Ref. [3] must also contain $\hat{I}_{\text{topological}}$ (See footnotes 2 and 3). Namely λ must appear in the form $\lambda = \frac{\kappa^2}{2} (\mathcal{V}_{\text{scalar}} + \hat{I}_{\text{brane}} + \hat{I}_{\text{topological}})$ in the Eq.(3.44) of Ref. [3]. But again, the omission of $\hat{I}_{\text{topological}}$ will not change the result of Ref. [3] at all because this $\hat{I}_{\text{topological}}$ is always gauged away (together with $\mathcal{V}_{\text{scalar}}$ and \hat{I}_{brane}) by \mathcal{E}_{SB} as described in Ref. [3] or Sec. 7.2 of this paper.

and $\delta_Q \hat{I}_{\text{brane}}^{(R)}$ arise from ρ_{vac} and $\delta\mu$, and they represent quantum fluctuations of the gravitational and standard model degrees of freedom with support on the $D3$ -branes. So $\delta_Q \hat{I}_{\text{brane}}^{(NS)} + \delta_Q \hat{I}_{\text{brane}}^{(R)}$ corresponds to the gravitational plus electroweak and QCD vacuum energies of the standard model configurations of the brane region.

The last term \mathcal{E}_{SB} is a supersymmetry breaking term, which originates from a gauge symmetry breaking of $A_{(4)}$ arising at the quantum level in the brane region. (See Sec.3.4 and 3.5 of Ref. [3] for the details.) \mathcal{E}_{SB} takes the form

$$\mathcal{E}_{\text{SB}} = - \int d^6 y \sqrt{h_6} \delta\mu_T^m(\Phi) f_m(y) \delta^6(y) , \quad (7.10)$$

where $\delta\mu_T^m$ represent quantum excitations on the brane with components along the transverse directions of the $D3$ -branes and $f_m(y)$ are arbitrary functions of y^m representing (derivatives of) local gauge parameters. Since \mathcal{E}_{SB} contains the gauge parameters $f_m(y)$, it has its own gauge arbitrariness.

Among the five terms in (7.9), $\hat{I}_{\text{brane}}^{(NS)}(\text{tree}) + \hat{I}_{\text{brane}}^{(R)}(\text{tree})$ vanishes for the BPS branes as mentioned above. But the quantum fluctuations $\delta_Q \hat{I}_{\text{brane}}^{(NS)} + \delta_Q \hat{I}_{\text{brane}}^{(R)}$ on the branes do not vanish when supersymmetry of the brane region is broken, though they are conjectured to cancel out to all orders of perturbations in the supersymmetric theories. So at the quantum level, \hat{I}_{brane} contained in (7.8) acquires nonzero contributions from these $\delta_Q \hat{I}_{\text{brane}}^{(NS)} + \delta_Q \hat{I}_{\text{brane}}^{(R)}$ and \mathcal{E}_{SB} in (7.10). Similarly, $\hat{I}_{\text{topological}}$ also acquires nonzero contributions only from the quantum corrections $\delta_Q \hat{I}_{\text{topological}}$ as we will see in Sec.8.2. $\hat{I}_{\text{topological}}$ is proportional to the $D3$ -brane potential $\Phi_-(y)$ (see (8.11)) and hence it vanishes at the ISD (tree level) background: $\hat{I}_{\text{topological}}(\text{tree}) = 0$. So λ in (7.8) finally reduces to

$$\lambda = \frac{\kappa^2}{2} \left(\frac{\beta_Q}{\beta} \right)^2 \delta_Q \hat{I}_{\text{total}} - \frac{1}{8} \Delta \mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta) , \quad (7.11)$$

where $\delta_Q \hat{I}_{\text{total}}$ represents

$$\delta_Q \hat{I}_{\text{total}} = \mathcal{V}_{\text{scalar}} + \delta_Q \hat{I}_{\text{brane}}^{(NS)} + \delta_Q \hat{I}_{\text{brane}}^{(R)} + \delta_Q \hat{I}_{\text{topological}} - \mathcal{E}_{\text{SB}} . \quad (7.12)$$

In (7.11), λ contains \mathcal{E}_{SB} which possesses its own gauge arbitrariness. So the fluctuations $\delta_Q \hat{I}_{\text{brane}}^{(NS)} + \delta_Q \hat{I}_{\text{brane}}^{(R)}$ in \hat{I}_{brane} , and also the remaining terms $\mathcal{V}_{\text{scalar}}$, $\delta_Q \hat{I}_{\text{topological}}$ and $\Delta \mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ are all gauged away (compensated) by \mathcal{E}_{SB} so that λ always satisfies the constraint equation (7.6).

VIII. Vanishing tree level actions

In this section we will show that the actions \hat{I}_{bulk} (or $\mathcal{V}_{\text{scalar}}$), \hat{I}_{brane} and $\hat{I}_{\text{topological}}$ all

vanish at the tree level : i.e. we have

$$\hat{I}_{\text{bulk}}(\text{tree}) = \hat{I}_{\text{brane}}(\text{tree}) = \hat{I}_{\text{topological}}(\text{tree}) = 0 . \quad (8.1)$$

Indeed, the vanishing of the tree level actions was already discussed in [6] for the case $G_{(3)} = 0$. Both $\hat{I}_{\text{bulk}}(\text{tree})$ and $\hat{I}_{\text{brane}}(\text{tree})$ vanish by 6D Einstein equations and field equations for ξ and A . In this section we will show that this is also the case in the time-dependent models of this paper. \hat{I}_{brane} and \hat{I}_{bulk} (and even $\hat{I}_{\text{topological}}$) all vanish at the tree level even when $G_{(3)} \neq 0$.

8.1 6D field equations

We start with the 6D Einstein equations to show that $\hat{I}_{\text{bulk}}(\text{tree}) = 0$ and $\hat{I}_{\text{brane}}(\text{tree}) = 0$. To understand the inside story of (8.1) more concretely we introduce a definite (but general) ansatz for the 6D metric

$$ds_6^2 \equiv h_{mn}(y)dy^m dy^n = dr^2 + R^2(r)d\Sigma_{1,1}^2 , \quad (8.2)$$

as in [6], where

$$d\Sigma_{1,1}^2 = \frac{1}{9} \left(d\psi + \sum_{i=1}^2 \cos \theta_i d\phi_i \right)^2 + \sum_{i=1}^2 \frac{1}{6} \left(d\theta_i^2 + \sin^2 \theta_i d\phi_i^2 \right) \quad (8.3)$$

is an Einstein metric representing the base of the cone in the conifold metric. The 6D Einstein equations can be obtained from (4.7) as before. For the metric (8.2) the 6D Einstein equations take the forms [6]

$$\mathcal{L}_F + c\chi^{-1/2} - 20 \left(\frac{R'^2}{R^2} - \frac{1}{R^2} \right) = 0 , \quad (8.4)$$

$$8 \frac{R''}{R} + \mathcal{L}_F - c\chi^{-1/2} + 12 \left(\frac{R'^2}{R^2} - \frac{1}{R^2} \right) = 0 , \quad (8.5)$$

where the "prime" denotes the derivative with respect to r . (8.4) and (8.5) are the rr and $\theta_i \theta_i$ components of the Einstein equation and there is no any other independent equation besides these two.

Besides the above Einstein equations, we also need the equations for $\xi(r)$ and $\chi^{1/2}(r)$. The equations of these fields can be found from the 6D total effective action $I_{\text{total}}^{(6D)}$ defined by

$$I_{\text{total}}^{(6D)} \equiv (I_{\text{IIB}} + I_{\text{brane}}) / \left(\int d^4 \hat{x} \sqrt{-\hat{g}_4} \frac{\alpha^4}{\beta^4} \Gamma_R^2 \right) , \quad (8.6)$$

where I_{IIB} and I_{brane} are given by (4.7) and (5.4). Using (5.1) one can rewrite (8.6) as

$$I_{\text{total}}^{(6D)} = \frac{1}{2\kappa_{10}^2 g_s^2} \int d^6 y \sqrt{h_6} \left(\mathcal{R}_6(h_{mn}) - \mathcal{L}_F + c\chi^{-1/2} \right) + \text{topological term}$$

$$+ \int d^6 y \sqrt{h_6} \left(-\frac{T_0}{g_s} \chi^{1/2} l(\gamma(\hat{t})) + \mu_0 \xi \right) \delta^6(y) , \quad (8.7)$$

where \mathcal{L}_F and the topological term are given by (4.8) and (4.10), respectively. $I_{\text{total}}^{(6D)}$ in (8.7) precisely coincides with the corresponding 6D effective action of the time-independent theory in [3] if $l(\gamma(\hat{t})) = 1$. In this section we are essentially looking for the classical (tree level) equations and therefore the tree level action in which $l(\gamma(\hat{t}))$ is given by $l(\gamma(\hat{t})) = 1$ is good enough to obtain the field equations we want. Now one can show that the field equation for ξ obtained from (8.7) coincides with the time-independent equation (4.5). Also, varying (8.7) with respect to A we obtain (see Sec. 7.2 of Ref. [3])

$$\begin{aligned} \nabla^2 \left(\frac{\chi^{1/2}}{g_s} \right) &= \frac{i}{12 \text{Im} \tau} \chi \mathcal{G}_{mnp} (*_6 \bar{\mathcal{G}})^{mnp} + \frac{1}{6 \text{Im} \tau} \chi \mathcal{G}_{mnp}^+ \bar{\mathcal{G}}^{+mnp} + \left(\frac{\chi^{1/2}}{g_s} \right)^{-1} \left[\partial \left(\frac{\chi^{1/2}}{g_s} \right) \right]^2 \\ &+ \left(\frac{\chi^{1/2}}{g_s} \right)^{-1} (\partial \xi)^2 + \frac{c}{g_s} + 2\kappa_{10}^2 T_0 \chi \delta^6(y) . \end{aligned} \quad (8.8)$$

8.2 Vanishing tree level actions

So far we have obtained four linearly independent equations (i.e. Eqs. (8.4), (8.5), (8.8) and (4.5)) defined on the 6D internal sector. In the followings we will use these equations to show that the vanishing of tree level actions in (8.1) is really the case. Now we start with the 6D Einstein equations in (8.4) and (8.5). These two equations can be solved by

$$c = 0 , \quad R(r) = r , \quad \mathcal{L}_F = 0 , \quad (8.9)$$

and from (8.9) one finds that \hat{I}_{bulk} in (5.3) vanishes at the tree level by the following reasons. By the solution $R(r) = r$ in (8.9), the unwarped metric (8.2) becomes the conifold metric $ds_{\text{conifold}}^2 = dr^2 + r^2 d\Sigma_{1,1}^2$. Hence at the tree level $\mathcal{R}_6(h_{mn})$ in (5.3) vanishes because the conifold metric is Ricci-flat. Also since $\mathcal{L}_F = 0$ as shown in (8.9), we find that \hat{I}_{bulk} in (5.3) vanishes at the tree level as mentioned above.

We turn to the equations $\hat{I}_{\text{brane}}(\text{tree}) = 0$ and $\hat{I}_{\text{topological}}(\text{tree}) = 0$. We see that $\hat{I}_{\text{brane}}(\text{tree})$ in (5.7) can be rewritten (for $\mu_0 = T_0$) as

$$\hat{I}_{\text{brane}}(\text{tree}) = -\frac{T_0}{g_s} \int d^6 y \sqrt{h_6} \Phi_-(y) \delta^6(y) , \quad (8.10)$$

and similarly from (4.10)

$$\hat{I}_{\text{topological}} = \frac{i}{4\kappa_{10}^2 \text{Im} \tau} \int \frac{\Phi_-(y)}{g_s} \mathcal{G}_3 \wedge \bar{\mathcal{G}}_{(3)} , \quad (8.11)$$

where $\Phi_-(y)$ is the D3-brane potential defined by (3.8). Now $\hat{I}_{\text{brane}}(\text{tree}) = 0$ and $\hat{I}_{\text{topological}}(\text{tree}) = 0$ can be shown from the two remaining equations (4.5) and (8.8) as

follows. Subtracting (4.5) from (8.8) (and also setting $\mu_0 = T_0$ again) one obtains

$$\nabla^2 \Phi_- = \frac{g_s}{6Im\tau} \chi |\mathcal{G}_{(3)}^+|^2 + \chi^{-1/2} |\partial \Phi_-|^2 + c , \quad (8.12)$$

where the last term c will vanish by (8.9).⁵ Equation (8.12) shows that the IASD fluxes $\mathcal{G}_{(3)}^+$ become a source for the potential Φ_- . But these IASD fluxes acquire nonzero values only at the quantum level and therefore $\mathcal{G}_{(3)}^+$ and Φ_- vanish in the ISD (i.e. tree level) background: $\mathcal{G}_-^{(0)} = \Phi_-^{(0)} = 0$, where $\mathcal{G}_- \equiv -i\mathcal{G}_{(3)}^+$. (See Sec. 2.3 of Ref. [8].) So we find that (8.10) and (8.11) both vanish because they are proportional to $\Phi_-(y)$, and therefore we have $\hat{I}_{\text{brane}}(\text{tree}) = \hat{I}_{\text{topological}}(\text{tree}) = 0$ together with $\hat{I}_{\text{bulk}}(\text{tree}) = 0$ as stated in (8.1).

We summarize the above results as follows. The tree level actions $\hat{I}_{\text{bulk}}(\text{tree})$, $\hat{I}_{\text{brane}}(\text{tree})$ and $\hat{I}_{\text{topological}}(\text{tree})$ all vanish: $\hat{I}_{\text{bulk}}(\text{tree}) = \hat{I}_{\text{brane}}(\text{tree}) = \hat{I}_{\text{topological}}(\text{tree}) = 0$ and therefore \hat{I}_{total} in (5.9) receives nonzero contributions only at the quantum level:

$$\hat{I}_{\text{total}} = \delta_Q \hat{I}_{\text{bulk}} + \delta_Q \hat{I}_{\text{brane}} + \delta_Q \hat{I}_{\text{topological}} . \quad (8.13)$$

In (8.13), $\delta_Q \hat{I}_{\text{brane}}$ represents $\delta_Q \hat{I}_{\text{brane}}^{(NS)} + \delta_Q \hat{I}_{\text{brane}}^{(R)} - \mathcal{E}_{\text{SB}}$ as described in Sec. 7.2 and similarly $\delta_Q \hat{I}_{\text{bulk}} / \delta_Q \hat{I}_{\text{topological}}$ are the contributions to $\hat{I}_{\text{bulk}} / \hat{I}_{\text{topological}}$ coming from the quantum corrections. In the case of the bulk action, $\delta_Q \hat{I}_{\text{bulk}}$ can be written in the form of a scalar potential $\mathcal{V}_{\text{scalar}}$ as follows. Using (7.6) one finds from (6.11) that

$$\hat{I}_{\text{bulk}} = \frac{1}{4\kappa_{10}^2 g_s^2} \int d^6 y \sqrt{h_6} (\mathcal{N} - 1) V , \quad (8.14)$$

and for $V \in V_3$, this becomes

$$\delta_Q \hat{I}_{\text{bulk}} = \mathcal{V}_{\text{scalar}} , \quad (8.15)$$

where we have used $\hat{I}_{\text{bulk}}(\text{tree}) = 0$ and therefore $\hat{I}_{\text{bulk}} = \delta_Q \hat{I}_{\text{bulk}}$. Originally, \hat{I}_{bulk} was defined by (5.3) and in our discussions we did not restrict it only to on-shell. Indeed, on the right hand side of (8.15) the main contributions to $\mathcal{V}_{\text{scalar}}$ are off-shell contributions coming from perturbative and nonperturbative corrections. (Note that V in $\mathcal{V}_{\text{scalar}}$ is given by $\propto \mathcal{G}_{(3)}^+ \cdot \bar{\mathcal{G}}_{(3)}^+$.) Collecting all these together, we find that \hat{I}_{total} in (8.13) can be written as

$$\hat{I}_{\text{total}} = \mathcal{V}_{\text{scalar}} + \delta_Q \hat{I}_{\text{brane}}^{(NS)} + \delta_Q \hat{I}_{\text{brane}}^{(R)} + \delta_Q \hat{I}_{\text{topological}} - \mathcal{E}_{\text{SB}} \equiv \delta_Q \hat{I}_{\text{total}} , \quad (8.16)$$

which coincides with (7.12).

IX. 4D equations of motion for the time dependent scale factors

⁵Note that $c = 0$ in (8.9) coincides with the constraint equation $c = 0$ in Sec. VII.

In this section we want to find 4D equations of motion for the time dependent scale factors \mathcal{A} , β_Q and β . To obtain these equations we go back to the 4D effective action (5.19). Using (5.14) and (5.17) one can rewrite (5.19) as

$$I_{\text{total}} = \frac{1}{2\kappa^2} \int d^3\vec{x} \int dt \mathcal{L}_{\text{total}} + \text{surface terms} , \quad (9.1)$$

where the total Lagrangian $\mathcal{L}_{\text{total}}$ is given by

$$\mathcal{L}_{\text{total}} = \mathcal{A}^3 \left(-6 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 + 2 \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + 6 \left(\frac{\dot{\beta}}{\beta} \right)^2 \right) + 2\kappa^2 \mathcal{A}^3 \frac{\beta_Q^2}{\beta^2} \delta_Q \hat{I}_{\text{total}} , \quad (9.2)$$

where we have used the relation $\hat{I}_{\text{total}} = \delta_Q \hat{I}_{\text{total}}$ in (8.16). The equations of motion following from (9.2) are

$$\frac{d}{dt} \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right) + \frac{3}{2} \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 + \frac{1}{2} \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + \frac{3}{2} \left(\frac{\dot{\beta}}{\beta} \right)^2 = -\frac{\kappa^2}{2} \frac{\beta_Q^2}{\beta^2} \delta_Q \hat{I}_{\text{total}} , \quad (9.3)$$

$$\frac{d}{dt} \left(\frac{\dot{\beta}_Q}{\beta_Q} \right) + 3 \frac{\dot{\mathcal{A}}}{\mathcal{A}} \frac{\dot{\beta}_Q}{\beta_Q} = \kappa^2 \frac{\beta_Q^2}{\beta^2} \delta_Q \hat{I}_{\text{total}} , \quad (9.4)$$

$$\frac{d}{dt} \left(\frac{\dot{\beta}}{\beta} \right) + 3 \frac{\dot{\mathcal{A}}}{\mathcal{A}} \frac{\dot{\beta}}{\beta} = -\frac{\kappa^2}{3} \frac{\beta_Q^2}{\beta^2} \delta_Q \hat{I}_{\text{total}} . \quad (9.5)$$

Besides these equations, we also have a constraint equation $\mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta) = 0$ which follows from $c = 0$ (see (6.9) and Sec. 7.1). Since $\mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta)$ is defined by (5.18), $\mathcal{R}_4^{(\text{eff})}(g_{\mu\nu}, \beta_Q, \beta) = 0$ implies that

$$6 \frac{d}{dt} \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right) + 6 \frac{d}{dt} \left(\frac{\dot{\beta}_Q}{\beta_Q} \right) + 12 \frac{d}{dt} \left(\frac{\dot{\beta}}{\beta} \right) + 12 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 + 2 \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + 6 \left(\frac{\dot{\beta}}{\beta} \right)^2 + 18 \frac{\dot{\mathcal{A}}}{\mathcal{A}} \frac{\dot{\beta}_Q}{\beta_Q} + 36 \frac{\dot{\mathcal{A}}}{\mathcal{A}} \frac{\dot{\beta}}{\beta} = 0 . \quad (9.6)$$

Equation (9.6) may be regarded as the last independent equation continued from the three equations in (9.3), (9.4) and (9.5). However, instead of (9.6), one can consider a different (i.e. a substitute) equation if he want. Namely a linear combination of (9.3), (9.4) and (9.5) gives

$$\begin{aligned} 6 \frac{d}{dt} \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right) + 6 \frac{d}{dt} \left(\frac{\dot{\beta}_Q}{\beta_Q} \right) + 12 \frac{d}{dt} \left(\frac{\dot{\beta}}{\beta} \right) + 9 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 + 3 \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + 9 \left(\frac{\dot{\beta}}{\beta} \right)^2 + 18 \frac{\dot{\mathcal{A}}}{\mathcal{A}} \frac{\dot{\beta}_Q}{\beta_Q} + 36 \frac{\dot{\mathcal{A}}}{\mathcal{A}} \frac{\dot{\beta}}{\beta} \\ = -\kappa^2 \frac{\beta_Q^2}{\beta^2} \delta_Q \hat{I}_{\text{total}} , \end{aligned} \quad (9.7)$$

and subtracting (9.7) from (9.6) one obtains

$$3 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 - \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 - 3 \left(\frac{\dot{\beta}}{\beta} \right)^2 = \kappa^2 \frac{\beta_Q^2}{\beta^2} \delta_Q \hat{I}_{\text{total}} , \quad (9.8)$$

which can be used as a substitute for (9.6) in our following discussions.

Equation (9.8) is much simpler than (9.6) and convenient for the later analysis. By this replacement the set of 4D equations is now given by (9.3), (9.4), (9.5) and the substitute equation (9.8). We rewrite them below for reader's convenience.

$$2\frac{d}{dt}\left(\frac{\dot{\mathcal{A}}}{\mathcal{A}}\right) + 3\left(\frac{\dot{\mathcal{A}}}{\mathcal{A}}\right)^2 + \left(\frac{\dot{\beta}_Q}{\beta_Q}\right)^2 + 3\left(\frac{\dot{\beta}}{\beta}\right)^2 = -Q , \quad (9.9)$$

$$\frac{d}{dt}\left(\frac{\dot{\beta}_Q}{\beta_Q}\right) + 3\frac{\dot{\mathcal{A}}}{\mathcal{A}}\frac{\dot{\beta}_Q}{\beta_Q} = Q , \quad (9.10)$$

$$\frac{d}{dt}\left(\frac{\dot{\beta}}{\beta}\right) + 3\frac{\dot{\mathcal{A}}}{\mathcal{A}}\frac{\dot{\beta}}{\beta} = -\frac{Q}{3} , \quad (9.11)$$

$$3\left(\frac{\dot{\mathcal{A}}}{\mathcal{A}}\right)^2 - \left(\frac{\dot{\beta}_Q}{\beta_Q}\right)^2 - 3\left(\frac{\dot{\beta}}{\beta}\right)^2 = Q , \quad (9.12)$$

where Q is a constant defined by

$$Q = \kappa^2 \frac{\beta_Q^2}{\beta^2} \delta_Q \hat{I}_{\text{total}} , \quad (9.13)$$

and where β and β_Q represent their present values of our universe as before.

X. Cosmological constant λ and evolving internal dimensions

10.1 The fine-tuning $\lambda = 0$

In the previous section we obtained a set of 4D equations (i.e. the equations from (9.9) to (9.12)) which must be satisfied by the present values of $\frac{\dot{\mathcal{A}}}{\mathcal{A}}$, $\frac{\dot{\beta}_Q}{\beta_Q}$, $\frac{\dot{\beta}}{\beta}$ and their derivatives. To solve these equations we introduce an usual ansatz for $\mathcal{A}(t)$ as

$$\mathcal{A}(t) = e^{Ht} \quad \left(H = \sqrt{\frac{\lambda}{3}} \right) , \quad (10.1)$$

where H is the Hubble constant of the present universe and in the absence of $T_{\mu\nu}$ of the matter fields it is only given by λ as in (10.1) (But see Sec. 10.2). Since λ (and therefore H^2) of our universe is almost vanishing in the units of Planck density, Q on the right hand sides of the equations must also be almost vanishing because from (9.9) and (9.12) it is expected to be of an order $\sim \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}}\right)^2 \sim \lambda$.

Equation (10.1) means that H is the present value of $\frac{\dot{A}}{A}$. Now we similarly introduce an ansatz for β_Q and β as

$$\beta_Q(t) = e^{q_0 t} , \quad \beta(t) = e^{b_0 t} , \quad (10.2)$$

which means that q_0 and b_0 are the present values of $\frac{\dot{\beta}_Q}{\beta_Q}$ and $\frac{\dot{\beta}}{\beta}$. Since $\frac{\dot{A}}{A}$ due to λ is constant, the first term of (9.9) vanishes:⁶

$$\frac{d}{dt} \left(\frac{\dot{A}}{A} \right) = 0 , \quad (10.3)$$

and the set of 4D equations from (9.9) to (9.12) reduces to

$$3H^2 + q_0^2 + 3b_0^2 = -Q , \quad (10.4)$$

$$d_Q + 3Hq_0 = Q , \quad (10.5)$$

$$d_B + 3Hb_0 = -\frac{Q}{3} , \quad (10.6)$$

$$3H^2 - q_0^2 - 3b_0^2 = Q , \quad (10.7)$$

where

$$d_Q \equiv \frac{d}{dt} \left(\frac{\dot{\beta}_Q}{\beta_Q} \right) , \quad d_B \equiv \frac{d}{dt} \left(\frac{\dot{\beta}}{\beta} \right) \quad (10.8)$$

represent their present values of our universe, respectively.

Apart from this, one can show that Q defined by (9.13) must be equal to λ . Eliminating $\triangle \mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ terms from (6.2) and (7.11) (and using $\hat{I}_{\text{total}} = \delta_Q \hat{I}_{\text{total}}$ of (8.16)) one can show that

$$Q = \lambda . \quad (10.9)$$

Now using $3H^2 = \lambda$ one finds that the only solution satisfying (10.4) to (10.7) together with (10.9) is

$$\lambda = 0 , \quad Q = 0 , \quad (10.10)$$

$$q_0 = 0 , \quad b_0 = 0 , \quad (10.11)$$

$$d_Q = 0 , \quad d_B = 0 . \quad (10.12)$$

Indeed this is the only solution. One can check that there is no other solution satisfying the set of above equations.

In the above solution $\lambda = 0$ in (10.10) is consistent with $q_0 = b_0 = 0$ in (10.11) by the following reason. First, $q_0 = b_0 = 0$ means that $\frac{\dot{\beta}_Q}{\beta_Q}$ and $\frac{\dot{\beta}}{\beta}$ vanish and in this case $\triangle \mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ also vanishes because $\triangle \mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ is given by (6.3). Next,

⁶This is indeed the case when $T_{\mu\nu}$ (besides $\lambda g_{\mu\nu}$) vanishes. (See Sec. IV of [9], for instance.)

$\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta) = 0$ implies $\lambda = 0$ by (7.6), so we see that $q_0 = b_0 = 0$ implies $\lambda = 0$ and therefore (10.10) is consistent with (10.11). Finally, one can check that $d_Q = d_B = 0$ in (10.12) is required by (10.5) and (10.6).

The fine-tuning $\lambda = 0$ (or $Q = 0$) in (10.10) can be achieved by \mathcal{E}_{SB} in $\delta_Q \hat{I}_{\text{total}}$ (see (7.12) or (8.16)). Since $\Delta\mathcal{R}_4(g_{\mu\nu}, \beta_Q, \beta)$ vanishes in the present case, (7.11) reduces to

$$\lambda = \frac{\kappa^2}{2} \left(\frac{\beta_Q}{\beta} \right)^2 \delta_Q \hat{I}_{\text{total}} , \quad (10.13)$$

(see also (9.13)) and therefore the fine-tuning $\lambda = 0$ ($Q = 0$) is equivalent to the requirement

$$\delta_Q \hat{I}_{\text{total}} = 0 , \quad (10.14)$$

where $\delta_Q \hat{I}_{\text{total}}$ is given by (7.12). But since $\delta_Q \hat{I}_{\text{total}}$ includes \mathcal{E}_{SB} and this \mathcal{E}_{SB} has its own gauge arbitrariness, the quantum corrections $\mathcal{V}_{\text{scalar}} + \delta_Q \hat{I}_{\text{brane}}^{(NS)} + \delta_Q \hat{I}_{\text{brane}}^{(R)} + \delta_Q \hat{I}_{\text{topological}}$ in $\delta_Q \hat{I}_{\text{total}}$ can be compensated by \mathcal{E}_{SB} so that $\delta_Q \hat{I}_{\text{total}}$, and therefore λ vanishes as a result.

In any case, the solution given by (10.10), (10.11) and (10.12) can be rewritten as

$$\lambda = 0, \quad \beta(t) = \beta_Q(t) = \gamma(t) = 1 , \quad (10.15)$$

which means that the internal dimensions (and also the dilaton e^Φ) do not evolve with time anymore. They remain fixed and at the same time the constraint equation $c = 0$ reduces to $\lambda = 0$. This result precisely coincides with the result of Ref. [3]. However, though the results coincide, the stabilization mechanism of this paper is entirely distinguished from the mechanism used in [3]. In [3], the internal dimensions are stabilized by a Kähler modulus-dependent nonperturbative correction of KKLT and therefore the no-scale structure of the scalar potential is broken as a result. But in this paper the internal dimensions are not stabilized by this KKLT scenario. The scale factor of the internal dimensions is stabilized by a set of 4D equations of the external sector which has nothing to do with the nonperturbative correction of the KKLT scenario. So the no-scale structure remains unbroken in our scenario of this paper and this may provide a new type of stabilization mechanism distinguished from the conventional KKLT.

10.2 Nonzero λ and evolving internal dimensions

So far we have discussed a new type of self-tuning (and a stabilization) mechanism in which the internal dimensions are basically allowed to evolve with time and in that case λ acquires nonzero contributions from the kinetic energies of this dynamical evolution of the internal dimensions (see (7.6)). So if the internal dimensions are static, then λ vanishes by (7.6) and the situation reduces back to the case of [3]. Indeed in Sec. 10.1 we obtained the fine-tuning $\lambda = 0$ with fixed internal dimensions in the framework of

the type IIB supergravity. In this case a set of 4D equations requires $\beta_Q(t) = \beta(t) = 1$ together with $\lambda = 0$. In the present section, however, we want to check the possibility of having nonzero λ (and also having evolving internal dimensions as well) by considering the stringy effects of the string theory.

The full string theory requires the action (2.1) to admit α' -corrections that are usually higher order in derivatives (see for instance [10, 11]). These terms have many (contracted) indices and therefore do not belong to V_n with $n = 1$ or 3 . If we denote the collection of these terms by ΔV , the scalar potential density now becomes

$$V = V_3 + \Delta V , \quad (10.16)$$

where V_3 represents the $\mathcal{G}_{mnp}^+ \bar{\mathcal{G}}^{+mnp}$ term which already exists in the supergravity Lagrangian \mathcal{L}_F . Since this V_3 is projected out by the operators in (7.3), it does not contribute to c as we already know. But the correction terms in ΔV do not satisfy this property. So in the presence of α' -corrections $c = 0$ is not the case anymore. c now takes nonzero values

$$c = \frac{1}{6} \chi^{1/2} (\mathcal{N} - 1) ((\mathcal{N} - 3)(1 - 3b_0 \Pi(\mathcal{N})) \Delta V , \quad (10.17)$$

and as a result of this (9.6) (which was obtained from (6.9) and $c = 0$) should be corrected into

$$\begin{aligned} 6 \frac{d}{dt} \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right) + 6 \frac{d}{dt} \left(\frac{\dot{\beta}_Q}{\beta_Q} \right) + 12 \frac{d}{dt} \left(\frac{\dot{\beta}}{\beta} \right) + 12 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 + 2 \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + 6 \left(\frac{\dot{\beta}}{\beta} \right)^2 \\ + 18 \frac{\dot{\mathcal{A}} \dot{\beta}_Q}{\mathcal{A} \beta_Q} + 36 \frac{\dot{\mathcal{A}} \dot{\beta}}{\mathcal{A} \beta} = \left(\frac{\beta_Q}{\beta} \right)^2 c . \end{aligned} \quad (10.18)$$

Besides this, there are also important changes in the equations (8.16) and (10.9). Since (6.11) does not imply (8.14) in the case $c \neq 0$, (8.15) should be changed (for $V \in V_3$) into

$$\delta_Q \hat{I}_{\text{bulk}} = \mathcal{V}_{\text{scalar}} - \frac{3c}{4\kappa^2} , \quad (10.19)$$

and therefore $\hat{I}_{\text{total}} = \delta_Q \hat{I}_{\text{total}}$ in (8.16) should also be changed into

$$\hat{I}_{\text{total}} = \delta_Q \hat{I}_{\text{total}} - \frac{3c}{4\kappa^2} , \quad (10.20)$$

in the case $c \neq 0$. So using (10.20), we find from (6.2) and (7.11) that (10.9) must be changed into

$$\lambda = Q - \frac{1}{4} \left(\frac{\beta_Q}{\beta} \right)^2 c . \quad (10.21)$$

Now we go back to the 4D equations of motion in (9.3), (9.4) and (9.5). These equations were obtained from the Lagrangian (9.2). But in the case $c \neq 0$, \hat{I}_{total} is given

by (10.20) instead of (8.16). So the Lagrangian (9.2) needs correction because it was obtained from the uncorrected equation (8.16). Using (10.20) and (6.8) we rewrite (5.19) as

$$I_{\text{total}} = -\frac{1}{4\kappa^2} \int d^3\vec{x} \int dt \mathcal{L}_{\text{total}} + \text{surface term} , \quad (10.22)$$

where $\mathcal{L}_{\text{total}}$ is now given by

$$\mathcal{L}_{\text{total}} = \mathcal{A}^3 \left(-6 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 + 2 \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + 6 \left(\frac{\dot{\beta}}{\beta} \right)^2 \right) - 4\kappa^2 \mathcal{A}^3 \frac{\beta_Q^2}{\beta^2} \delta_Q \hat{I}_{\text{total}} . \quad (10.23)$$

The corrected Lagrangian (10.23) almost coincides with the original Lagrangian (9.2). It differs from (9.2) only in that $\delta_Q \hat{I}_{\text{total}}$ in (9.2) is replaced by $-2\delta_Q \hat{I}_{\text{total}}$. So the equations following from (10.23) take the same forms as the original equations in (9.3), (9.4) and (9.5) only except that $\delta_Q \hat{I}_{\text{total}}$ in the equations is replaced by $-2\delta_Q \hat{I}_{\text{total}}$. This prescription also applies to (9.7). The corrected version of (9.7) would be

$$\begin{aligned} 6 \frac{d}{dt} \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right) + 6 \frac{d}{dt} \left(\frac{\dot{\beta}_Q}{\beta_Q} \right) + 12 \frac{d}{dt} \left(\frac{\dot{\beta}}{\beta} \right) + 9 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 + 3 \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + 9 \left(\frac{\dot{\beta}}{\beta} \right)^2 \\ + 18 \frac{\dot{\mathcal{A}} \dot{\beta}_Q}{\mathcal{A} \beta_Q} + 36 \frac{\dot{\mathcal{A}} \dot{\beta}}{\mathcal{A} \beta} = 2\kappa^2 \frac{\beta_Q^2}{\beta^2} \delta_Q \hat{I}_{\text{total}} , \end{aligned} \quad (10.24)$$

and subtracting (10.24) from (10.18) we obtain

$$3 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 - \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 - 3 \left(\frac{\dot{\beta}}{\beta} \right)^2 = \left(\frac{\beta_Q}{\beta} \right)^2 c - 2\kappa^2 \frac{\beta_Q^2}{\beta^2} \delta_Q \hat{I}_{\text{total}} , \quad (10.25)$$

which is corrected version of the substitute equation (9.8).

Collecting all these together, we can now write the corrected versions of the 4D equations in Sec. 10.1 as

$$2 \frac{d}{dt} \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right) + 3 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 + \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 + 3 \left(\frac{\dot{\beta}}{\beta} \right)^2 = 2Q , \quad (10.26)$$

$$\frac{d}{dt} \left(\frac{\dot{\beta}_Q}{\beta_Q} \right) + 3 \frac{\dot{\mathcal{A}} \dot{\beta}_Q}{\mathcal{A} \beta_Q} = -2Q , \quad (10.27)$$

$$\frac{d}{dt} \left(\frac{\dot{\beta}}{\beta} \right) + 3 \frac{\dot{\mathcal{A}} \dot{\beta}}{\mathcal{A} \beta} = \frac{2}{3} Q , \quad (10.28)$$

$$3 \left(\frac{\dot{\mathcal{A}}}{\mathcal{A}} \right)^2 - \left(\frac{\dot{\beta}_Q}{\beta_Q} \right)^2 - 3 \left(\frac{\dot{\beta}}{\beta} \right)^2 = -2Q + \left(\frac{\beta_Q}{\beta} \right)^2 c , \quad (10.29)$$

which, upon using (10.1) and (10.2), reduce into

$$3H^2 + q_0^2 + 3b_0^2 = 2Q , \quad (10.30)$$

$$d_Q + 3Hq_0 = -2Q , \quad (10.31)$$

$$d_B + 3Hb_0 = \frac{2}{3}Q , \quad (10.32)$$

$$3H^2 - q_0^2 - 3b_0^2 = -2Q + \left(\frac{\beta_Q}{\beta}\right)^2 c , \quad (10.33)$$

where the nonzero values of c are given by (10.17), while Q is still defined by (9.13). The above equations are the corresponding equations of (10.4), (10.5), (10.6) and (10.7) in Sec. 10.1, and similarly (10.21) of this section is the corresponding equation of (10.9) in Sec. 10.1.

Now we have five equations (Eqs. from (10.30) to (10.33) and Eq. (10.21)) which must be satisfied by the solution of this section. We solve these equations as follows. First, using $3H^2 = \lambda$ we recast the two equations in (10.30) and (10.33) into

$$\lambda = \frac{1}{2} \left(\frac{\beta_Q}{\beta}\right)^2 c , \quad (10.34)$$

and

$$q_0^2 + 3b_0^2 = 2Q - \frac{1}{2} \left(\frac{\beta_Q}{\beta}\right)^2 c . \quad (10.35)$$

Next, from (10.21) and (10.34) we obtain

$$Q = \frac{3}{4} \left(\frac{\beta_Q}{\beta}\right)^2 c , \quad (10.36)$$

and therefore (10.34) and (10.35) can be rewritten as

$$\lambda = \frac{2}{3}Q , \quad (10.37)$$

and

$$q_0^2 + 3b_0^2 = 2\lambda . \quad (10.38)$$

The constraint (10.37) (or equivalently (10.36)) can be achieved by \mathcal{E}_{SB} contained in Q (see (7.12) and (9.13)). Since \mathcal{E}_{SB} has gauge arbitrariness, $\mathcal{V}_{\text{scalar}} + \delta_Q \hat{I}_{\text{brane}}^{(NS)} + \delta_Q \hat{I}_{\text{brane}}^{(R)} + \delta_Q \hat{I}_{\text{topological}}$ in $\delta_Q \hat{I}_{\text{total}}$ can be gauged away by \mathcal{E}_{SB} so that Q adjusts itself to satisfy (10.36) or (10.37).

The above result shows that the equations of this section admit solutions with nonzero λ . (10.34), which is the generalization of (10.10) to the case $c \neq 0$, shows that λ must take nonzero values because c in (10.17) does so. In addition, (10.38) requires that λ must be positive because q_0 and b_0 are real. This positive λ , however, must be very small because it is of the same order as c by (10.34). For instance, if ΔV in (10.17) is given by the effective Lagrangian in [10], then c , and therefore λ must at least be of an order

α'^3 . Besides this, c also contains the function $\chi^{1/2}(y)$. In the simple compactifications with $G_{(3)} = 0$ this function takes the form [6]

$$\chi^{1/2}(r) = \left(1 + \frac{Q_0}{r^4}\right)^{-1}, \quad (Q_0 = \text{constant}), \quad (10.39)$$

and hence in the neighborhood of the brane at $r = 0$ it reduces to $\chi^{1/2}(r) \sim r^4/Q_0$, which shows c is highly suppressed again because $\chi^{1/2}(r)$ strongly vanishes at $r = 0$ ($y = 0$) in the approximation $G_{(3)} \cong 0$. In any case, λ given by (10.34) will be very small anyway.

We finally determine q_0 and b_0 from (10.31), (10.32) and (10.38). Using (10.37) and $\lambda = 3H^2$ we rewrite them as

$$\hat{q}_0 = -3 - \hat{d}_Q, \quad (10.40)$$

$$\hat{b}_0 = 1 - \hat{d}_B, \quad (10.41)$$

$$\hat{q}_0^2 + 3\hat{b}_0^2 = 6, \quad (10.42)$$

where \hat{d}_Q and \hat{d}_B are defined by

$$\hat{d}_Q \equiv \frac{d_Q}{3H^2}, \quad \hat{d}_B \equiv \frac{d_B}{3H^2}, \quad (10.43)$$

and similarly

$$\hat{q}_0 \equiv \frac{q_0}{H}, \quad \hat{b}_0 \equiv \frac{b_0}{H}. \quad (10.44)$$

The set of equations from (10.40) to (10.42) allows for infinite numbers of solutions. A unique and the simplest solution that comes to our mind would be obtained by considering an ansatz

$$\hat{d}_Q = -3\hat{d}_B. \quad (10.45)$$

The solution satisfying this ansatz is

$$\hat{q}_0 = -\frac{3}{\sqrt{2}}, \quad \hat{b}_0 = \frac{1}{\sqrt{2}}, \quad (10.46)$$

with

$$\hat{d}_Q = -3\left(1 - \frac{1}{\sqrt{2}}\right), \quad \hat{d}_B = 1 - \frac{1}{\sqrt{2}}. \quad (10.47)$$

To see why this solution is unique, we go back to (5.10) and use (10.2) to rewrite it as

$$\gamma(t) = e^{(3\hat{b}_0 + \hat{q}_0)Ht}. \quad (10.48)$$

In (10.48), $\gamma(t)$ reduces to $\gamma(t) = 1$ by (10.45), which means that the string coupling (or the dilaton) is given by the time-independent form

$$e^{\Phi(y,t)} = g_s e^{\Phi_s(y)}, \quad (10.49)$$

despite that the scale factor $\beta(t)$ of the internal dimensions still remains time-dependent in the above solution. Indeed we have

$$\beta(t) = e^{\frac{1}{\sqrt{2}}Ht} \quad (10.50)$$

from (10.2) and (10.46), which suggests that the internal dimensions are now under accelerated expansion almost at the same rate as the external dimensions.

XI. Summary and discussion

In this paper we presented a new scenario for the moduli stabilization and for a very small but nonzero positive λ in the framework of the self-tuning mechanism proposed in [3]. In our scenario the complex structure moduli are still stabilized by the three-form fluxes as in the usual flux compactifications. But the Kähler modulus of the internal dimensions is not fixed by the usual KKLT-type mechanism. In our paper we assumed that the scale factor of the internal dimensions is basically allowed to evolve with time. But at the supergravity level the result of our scenario of this paper precisely coincides with the result of the time-independent theory presented in [3]. We obtained $\lambda = 0$ and also fixed internal dimensions with $\beta(t) = 1$ as in [3].

Though the results coincide the stabilization mechanism of this paper is very distinguished from the mechanism used in [3]. In [3], the internal dimensions are stabilized by a Kähler modulus-dependent nonperturbative correction of KKLT and therefore the no-scale structure of the Lagrangian is broken in that case. But in this paper the internal dimensions are not stabilized by this KKLT scenario. The Kähler modulus of the internal dimensions is stabilized by a set of 4D dynamical equations defined on the external spacetime and the no-scale structure is unbroken in our scenario of this paper.

The above result changes once we admit α' -corrections of the string theory. Namely $\lambda = 0$ changes into a new fine-tuning $\lambda = \frac{2}{3}Q$ (Eq. (10.37)), where Q is related to the constant c by the equation $Q = \frac{3}{4}\left(\frac{\beta_Q}{\beta}\right)^2 c$ (Eq. (10.36)), and where c takes nonzero values arising from the α' -corrections (see (10.17)). So λ in (10.37) acquires nonzero values from the α' -corrections and in the limit $\alpha' \rightarrow 0$ it reduces back to $\lambda = 0$ as it should be. But in any case, the fine-tunings of either $\lambda = 0$ or $\lambda = \frac{2}{3}Q$ can be achieved by the supersymmetry breaking term \mathcal{E}_{SB} contained in Q (see (9.13) and (7.12)). Namely, for a given value of c (which is determined from the α' -corrections) \mathcal{E}_{SB} adjust itself (recall that \mathcal{E}_{SB} has gauge arbitrariness) so that Q satisfies (10.36), and this nonzero Q becomes a nonzero λ by (10.37).

In the case of nonzero λ (i.e. when we admit α' -corrections) the internal dimensions generically evolve with time as opposed to the case $\lambda = 0$. Indeed the set of 4D equations

requires that the scale factor of the internal dimensions must be of the form $\beta(t) = e^{\hat{b}_0 H t}$, where \hat{b}_0 denotes dimensionless constants of order one. Among these solutions, of particular interest is the one that given by $\beta(t) = e^{\frac{1}{\sqrt{2}} H t}$ (Eq. (10.50)). This solution corresponds to $\gamma(t) = 1$, which means that the string coupling e^Φ of this solution remains time-independent though the internal dimensions evolve with time. Apart from this particular case, $\beta(t)$ and $\gamma(t)$ are generically nontrivial functions of time in the case of nonzero λ .

We finally discuss the observational effects of the time-evolving $\beta(t)$ and $\gamma(t)$. Since $\beta(t)$ is given by $\beta(t) = e^{\hat{b}_0 H t}$ with \hat{b}_0 being of order one, the internal dimensions must expand almost at the same rate as the 4D external spacetime and such an expansion of the internal dimensions may lead to time-varying constants of nature. For instance, it is well known [12] that the coupling constants g_c including electric charges are inversely proportional to the radius R of the compact internal dimensions:

$$g_c \sim \frac{\kappa}{R}, \quad (11.1)$$

where $\kappa \equiv \sqrt{16\pi G}$. Also in string theory the 4D gravitational constant G behaves like [13]

$$G \sim \alpha' e^{2\Phi}. \quad (11.2)$$

So the coupling constants $g_c(t)$ take the form

$$g_c(t) \sim g_s \alpha'^{1/2} e^{\Phi_s - \frac{B}{2}} \frac{\gamma(t)}{\beta(t)}, \quad (11.3)$$

and for the solution with $\beta(t) = e^{\frac{1}{\sqrt{2}} H t}$ and $\gamma(t) = 1$ they become

$$g_c(t) \sim g_c(0) e^{-\frac{1}{\sqrt{2}} H t}, \quad (11.4)$$

where $g_c(0) \equiv g_s \alpha'^{1/2} e^{\Phi_s - \frac{B}{2}}$.

The above results suggest that some of the well-known constants of nature might not be real constants. For instance, (11.2) shows that the gravitational constant G is proportional to $\gamma^2(t)$. So if choose the solutions with $\dot{\gamma}(t) \neq 0$, then G becomes a time-varying quantity.⁷ Similarly, (11.4) shows that the coupling constants including electric charges are also time-varying quantities. They are decreasing in magnitudes at the same rate as the expansions of the internal and external dimensions. The decreasing or expanding rate of these quantities is very small. It is about $\sim e^{Ht}$, where the Hubble constant H of our present universe is roughly given by [14]

$$H^{-1} \sim 10^{10} \text{ yr}. \quad (11.5)$$

⁷But if we choose $\gamma(t) = 1$, then G is still real constant.

So, for instance, the electric charges of our present universe decrease in magnitudes at the rate in which they become half the original magnitudes during about 10^{10} years, while the internal dimensions become doubled in size during that time. These results of our scenario might be checked by experiments. If they coincide, then it would be great. But if not, then we go back to Ref. [3] to see what happens to the self-tuning equation in [3] when we admit α' -corrections.

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